Chapter 2
Thermodynamic Fundamentals for Production of Electric Power in Hierarchical *j*-Cycle Systems

Abstract This chapter presents thermodynamic fundamentals for the generation of power in the hierarchical *j*-cycle systems.

Keywords Thermodynamic fundamentals · Hierarchical j-cycle systems

The highest theoretical efficiency of generating mechanical energy (and, as a consequence, electric energy) in thermodynamic systems can be achieved by adopting Carnot cycle—Fig. 2.1.

This efficiency is expressed by the equation:

$$\eta_C = 1 - \frac{T_{amb}}{T_h} \tag{2.1}$$

and the power output from Carnot heat engine by the equation:

$$N_C = \eta_C \dot{Q}_d \tag{2.2}$$

where:

$\dot{Q}_d$ stream of the driving heat,
T_{amb} absolute temperature of the cold reservoir (environment),
T_h absolute temperature of the hot reservoir.

The power N_C is equivalent with the exergy stream $\dot{B}$ of stream of heat $\dot{Q}_d$ transferred from the source with the temperature of $T_h = \text{const}$, $N_C \equiv \dot{B}_{\dot{Q}_d}$.

If a power plant were to realize the Carnot cycle (which is technically impossible), for the temperatures $T_h = T_{com} = 1600$ K and $T_{amb} = 300$ K its efficiency, under the assumption of a lack of losses during the conversion of mechanical energy into electric energy would be $\eta_C = 81$ % (T_{com} denotes the temperature of the combustion of coal in the boiler). Concurrently, the gross

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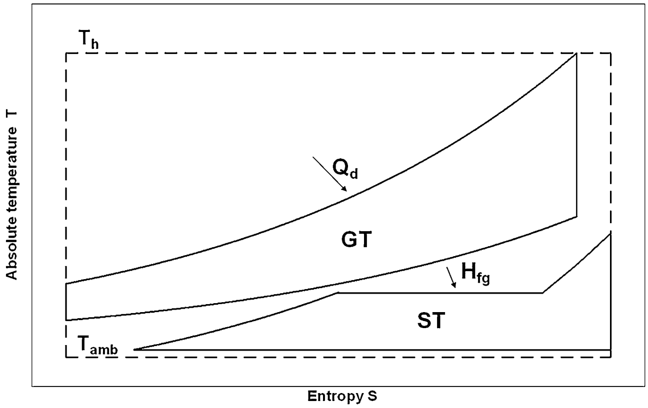


Fig. 2.1 Comparative cycle (theoretical) of a single-fuel gas–steam system (GT—Joule's cycle of the gas turbine, ST—Clausius-Rankine cycle of the steam turbine, Q_d—driving heat transferred into GT, H_{fg}—enthalpy of flue gas exiting from the gas turbine transferred to ST through heat recovery steam generator; *dashed line* marks Carnot cycle for the extreme temperatures T_{amb} and T_h)

efficiency of a steam based Clausius-Rankine cycle realized in a power station is smaller by around 50 %. For instance, for a 370 MW power unit operating under subcritical parameters this efficiency is equal to mere 41 %.

From the equation in (2.1), a conclusion can be made that the same stream of heat $\dot{Q}_d$ transferred from the source with the temperature of $T_h = \text{const}$ can be transformed into mechanical power to the greater degree the higher the value of temperature T_h. The power (stream of exergy) losses as a result of lowering the temperature from T_{h1} to T_{h2} ($T_{h1} > T_{h2}$) is equal to:

$$\Delta N_C = \delta \dot{B} = \dot{Q}_d \left(1 - \frac{T_{amb}}{T_{h\,1}}\right) - \dot{Q}_d \left(1 - \frac{T_{amb}}{T_{h\,2}}\right) = T_{amb} \dot{Q}_d \frac{T_{h\,1} - T_{h\,2}}{T_{h\,1} T_{h\,2}}. \tag{2.3}$$

The value on the right hand side of Eq. (2.3) concurrently denotes the loss of exergy stream in the irreversible heat transfer between the two sources with the temperatures of $T_{h1} = \text{const}$ and $T_{h2} = \text{const}$. This loss can be expressed also in terms of the increase of their entropy streams (compare Eq. 2.8).

By analogy to formula (2.1), the energy efficiency of any cycle can be expressed by the equation [1]:

$$\eta_t = 1 - \frac{\overline{T}_{out}}{\overline{T}_{in}} \tag{2.4}$$

where the temperature T_h of the isotherm of the Carnot cycle that is equal to the temperature of the hot reservoir can be replaced by the entropy averaged temperature $\overline{T}_{in}$ during the transfer of heat into a medium in an arbitrarily considered cycle (Eq. 2.5), and the temperature T_{amb} of the isotherm of the Carnot cycle of the cold

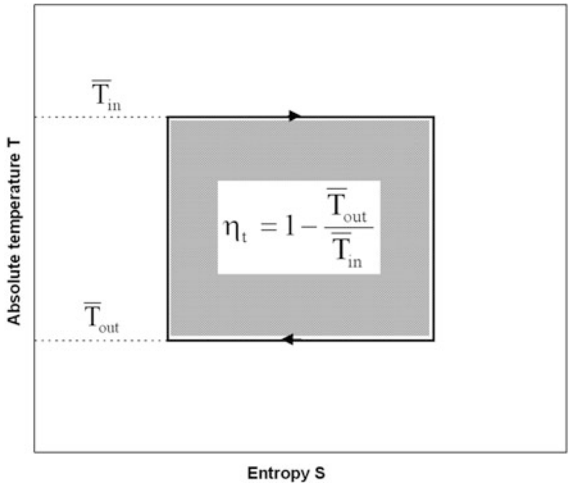


Fig. 2.2 Thermodynamic cycle

reservoir, i.e. the environment, is replaced by the entropy averaged temperature $\overline{T}_{out}$ during the extraction of heat from an arbitrary cycle—Fig. 2.2.

The introduction of mean thermodynamic temperatures $\overline{T}_{in}$ and $\overline{T}_{out}$ for the subsequent input and output of heat from a system (which are calculated for the actual temperatures and pressures at the beginning and output from these processes, i.e. for irreversible processes), makes it possible to present any cycle in a *temperature-entropy* co-ordinate system in the form of a rectangular shape (Fig. 2.2), regardless of the nature of the processes of the physical changes occurring during them, including those in which actual effective work is exerted, whether reversible or not. The exergy losses as a result of the friction during these conversions only have to be involved in the mean values of the entropy changes at temperatures of the heat input $\overline{T}_{in}$ and output $\overline{T}_{out}$ from a system only in the case if predefined correction measures are adopted. The quotient of the averaged temperatures during these conversions has to apparently equal to the quotient of the heat output Q_{out} and input Q_{in} into a given cycle [1].

From the relation in (2.4) it stems that the generation of electricity in the cycles of thermal power plants should be undertaken for a technically maximum temperature $\overline{T}_{in}$ of the circulating medium during the input of stored heat, i.e. heat from an external source and for the lowest temperature $\overline{T}_{out}$ of the medium output of heat from the cycle in a power plant.

Clausius-Rankine cycle is followed in a coal-fired power plants (Fig. 2.1). From the thermodynamic perspective, its fundamental drawback is associated with the low mean thermodynamic temperature $\overline{T}_{in}$ of the circulating media—water and steam (also called the entropy averaged temperature) during isobaric process of the heat Q_{in} transfer into this cycle in the boiler (compare Eqs. 2.6, 2.7):

$$\overline{T}_{in} = \frac{Q_{in}}{\Delta S} = \frac{\int_{s_w}^{s_s} T_{in}(s)ds}{s_s - s_w} = \frac{h_s - h_w}{s_s - s_w} \tag{2.5}$$

where:

ΔS increase of the entropy of the circulating medium,
h, s specific enthalpy and entropy of the circulating medium.

In a 370 MW power unit, the thermal parameters of the water fed into the boiler are equal to: 255 °C/23.5 MPa ($h_w = 1110.8$ kJ/kg, $s_w = 2.7947$ kJ/(kgK)), respectively the parameters of fresh steam are 535 °C/18 MPa ($h_s = 3373.2$ kJ/kg, $s_s = 6.3537$ kJ/(kgK); hence, the mean thermodynamic temperature is equal to only $\overline{T}_{in} = 635$ K (while accounting for inter-stage steam superheating $\overline{T}_{in} = 640.7$ K). Concurrently, the temperature of the combustion of coal in the boiler is equal to around $T_{com} = 1600$ K. Therefore, the temperature difference $T_{com} - \overline{T}_{in} \cong 1000\,K$ is considerable, which along with the low temperature $\overline{T}_{in}$ results in small efficiency of generating electricity in a power unit (from Eqs. 2.1–2.4) it stems that it is equal to mere 41 % gross (which is 37 % net).

The power losses expressed by the Eq. (2.3) takes place in the steam boiler, while $\dot{Q}_d = \dot{E}_{ch}^{coal}$ ($\dot{E}_{ch}^{coal}$ denotes the stream of chemical energy of the coal) is equal to the product of the stream of coal combustion in the boiler $\dot{P}$ and its net calorific value NCV, $\dot{Q}_d = \dot{E}_{ch}^{coal} = \dot{P}(NCV)$, and temperatures T_{h1} and T_{h2} are equal to: $T_{h1} = T_{com}$ and $T_{h2} = \overline{T}_{in}$. In terms of numbers, the loss of power is equal to 30 % of the driving heat $\dot{Q}_d$: $T_{amb}(T_{com} - \overline{T}_{in})/(T_{com}\overline{T}_{in}) = 300(1600 - 636)/(1600 \times 636) \cong 30\,\%$. Thus, despite its high energy efficiency, reaching 94 %, the steam boiler forms the major source of the low efficiency of generating electric power in steam power plants operating in the Clausius-Rankine cycle.

However, the considerable advantage of the Clausius-Rankine cycle is the low temperature $\overline{T}_{out}$ in it. The condensation isotherm in it nearly overlaps with the isotherm of the ambient temperature in the Carnot cycle, $\overline{T}_{out} \cong T_{amb}$ (Fig. 2.1).

The use of the higher range of temperature, even starting from the temperature of gas combustion $t_{com} = 1500$ °C is taken advantage of in gas turbogenerators. The production of electric power in it occurs by direct expansion of exhaust gas from the temperature and pressure in the combustion chamber to the ambient pressure. Hence, a coupling of the steam system with the gas system, whose advantage involves a considerably higher temperature $\overline{T}_{in}$ compared with the steam boiler (the disadvantage of the gas system involves also the high temperature $\overline{T}_{out}$ of the circulating medium during the extraction of heat from it), results in the use of the advantages of the two cycles while avoiding their drawbacks. As a result, the efficiency of producing electricity in power plants adapted to a gas-steam system considerably increases. The device which couples the two cycles is the heat recovery steam generator—Fig. 1.1b. The steam produced in it has

identical thermal parameters as the steam from the coal-fired boiler. The total stream of fresh steam from the heat recovery steam generator and coal-fired boiler is equal to the stream of steam prior to when the system was not repowered. The production of steam in the heat recovery steam generator applies the stream of low-temperature enthalpy of the flue gas H_{fg} from the gas turbine. Thus, its enthalpy partly replaces the use of coal in the existing coal-fired system, due to which the use of coal is limited. As a result, the loss of the unused higher range of temperatures $T_{com} - \overline{T}_{in} \cong 1000\,K$ is reduced. As an additional consequence, the efficiency of the generation of electricity in dual-fuel gas-steam systems is improved. This efficiency increases along with the increase of the capacity of the gas turbine and it can reach as much as by 10 %.

The highest efficiency (Eq. 2.19), even as much as 60 %, is possible in single-fuel gas-steam systems (Figs. 2.1, 2.4) [1], where the coal-fired boiler is excluded and, thus, the phenomenon of unused higher range of the temperature of the flue gas is avoided. The total driving heat Q_d from the combustion of gas (or liquid fuel; a very attractive concept in terms of energy and economic efficiency involves direct coal combustion in a gas turbine) is input into a gas turbine operating under the Joule's cycle (Fig. 2.1). The steam-based section still operates in the Clausius-Rankine cycle, but the driving heat for the production of steam originates only from the low-temperature enthalpy of the flue gas H_{fg} extracted from the gas turbine. As a result, the loss of the unused higher range of temperatures is avoided while the efficiency of the production of electricity increases in comparison to the system solely based on steam. Such an increase of efficiency can be explained in a form of a chart. As one can see in Fig. 2.1, the Carnot cycle is supplemented by the Joule's cycle, as a result of which there is a considerable reduction of the surface areas of the conversion phenomena in the Clausius-Rankine cycle and Carnot cycle.

In a general case, the number of circulating media can be arbitrarily large. A hierarchical j-cycle system is presented in Fig. 2.3. An increase of the number of media with various temperatures of the operating range makes it possible to apply in a system higher range of the temperature increase between the upper and lower heat sources (environment). Thereby, exergy losses in the system are reduced and the production of electricity increases. The disadvantage of such a solution includes an increase of investment required to start the system.

Generally, the loss of exergy stream $\delta\dot{B} = T_{amb}\left(\sum_k \Delta\dot{S}_{med} + \sum_l \Delta\dot{S}_{so}\right)$ in a hierarchical "j-cycle" system comes as a consequence of mere increase of entropy streams of external heat sources $\sum_l \Delta\dot{S}_{so}$ which are in contact with it [1] (in practice we usually have to do with two sources, $l = 2$). The substitution of actual open cycle processes by closed-loop system, which normally facilitates the thermodynamic analysis of such processes, leads to a lack of consideration of the media input and output from the system; hence, the increase of entropy streams is equal to zero, $\sum_k \Delta\dot{S}_{med} = 0$. Hence, an increase of the entropy of the bodies which participate in the phenomenon is expressed only in terms of the increase of the entropy of heat

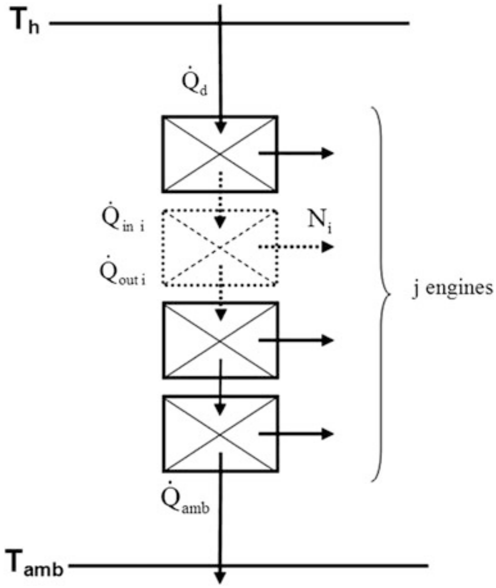


Fig. 2.3 Diagram of hierarchical *j*-cycle heat engine

sources. This increase can then be expressed in terms of the total of entropy increases in irreversible heat flow between the sources and cycles as well as between the cycles.

One can note in this place that the increase of entropy of the external source of heat with the temperature $T_{so} = \text{const}$ which delivers heat Q into the system can be derived from the definition of entropy

$$\Delta S_{so} = -\int \frac{dQ}{T_{so}} = -\frac{Q}{T_{so}}. \tag{2.6}$$

The minus sign in Eq. (2.6) denotes that the positive heat Q was extracted from the source. For the source which pulls heat Q from the system, it is only necessary in Eq. (2.6) to change the sign

$$\Delta S_{so} = +\int \frac{dQ}{T_{so}} = +\frac{Q}{T_{so}}. \tag{2.7}$$

By applying the Eqs. (2.6) and (2.7) the loss of exergy stream $\delta \dot{B}$ in a closed system with two heat sources with the temperatures of T_h and T_{amb} (Fig. 2.3) can be expressed by the equation [1] (compare Eq. 2.3):

$$\begin{aligned} \delta \dot{B} &= T_{amb} \sum_{2} \Delta \dot{S}_{so} = T_{amb}\left(\frac{\dot{Q}_{amb}}{T_{amb}} - \frac{\dot{Q}_d}{T_h}\right) = \sum_{i=1}^{j+1} \delta \dot{B}_i \\ &= T_{amb} \sum_{i=1}^{j+1} \dot{Q}_{in\ i} \frac{\overline{T}_{out\ i-1} - \overline{T}_{in\ i}}{\overline{T}_{out\ i-1} \overline{T}_{in\ i}}, \end{aligned} \tag{2.8}$$

and the capacity of the system by the equation:

$$
\begin{aligned}
N &= N_C - \delta\dot{B} = \dot{Q}_d \frac{T_h - T_{amb}}{T_h} - T_{amb}\left(\frac{\dot{Q}_{amb}}{T_{amb}} - \frac{\dot{Q}_d}{T_h}\right) \\
&= \sum_{i=1}^{j} N_i = \sum_{i=1}^{j} (\dot{Q}_{in\,i} - \dot{Q}_{out\,i}) = \sum_{i=1}^{j} \dot{Q}_{in\,i} \frac{\overline{T}_{in\,i} - \overline{T}_{out\,i}}{\overline{T}_{in\,i}} \\
&= \dot{Q}_d \frac{T_h - T_{amb}}{T_h} - T_{amb} \sum_{i=1}^{j+1} \dot{Q}_{in\,i} \frac{\overline{T}_{out\,i-1} - \overline{T}_{in\,i}}{\overline{T}_{out\,i-1}\overline{T}_{in\,i}}
\end{aligned}
\tag{2.9}
$$

where:

j	number of circulating media (engines),
N_c, N_i	capacity of a theoretical Carnot engine and actual engines,
$\dot{Q}_{in\,i}$, $\dot{Q}_{out\,i}$	heat of stream input into and output from an i-th cycle (engine), while $\dot{Q}_{out\,i} = \dot{Q}_{in\,i+1}$ and $\dot{Q}_{in\,1} \equiv \dot{Q}_d$, $\dot{Q}_{in\,j+1} \equiv \dot{Q}_{amb}$,
$\dot{Q}_{amb}$, $\dot{Q}_d$	stream of heat transmitted from the system into the environment and delivered from the upper source of heat,
$\overline{T}_{in\,i}$, $\overline{T}_{out\,i}$	mean thermodynamic temperature of the absorbing medium, and giving off heat in an i-th cycle (engine), while $\overline{T}_{in\,j+1} \equiv T_{amb}$, $\overline{T}_{out\,0} \equiv T_h$,
T_h	absolute temperature of the upper source of heat.

The value of $(\overline{T}_{in\,i} - \overline{T}_{out\,i})/\overline{T}_{in\,i}$ in the final term of the Eq. (2.9) represents the energy efficiency of an i-th ($i = 1 \div j$) engine operating between entropy averaged temperatures in actual processes of heat input and output $\overline{T}_{in\,i}$, $\overline{T}_{out\,i}$ from a system (compare Eq. 2.4, Fig. 2.2):

$$
\eta_i = 1 - \frac{\overline{T}_{out\,i}}{\overline{T}_{in\,i}} \tag{2.10}
$$

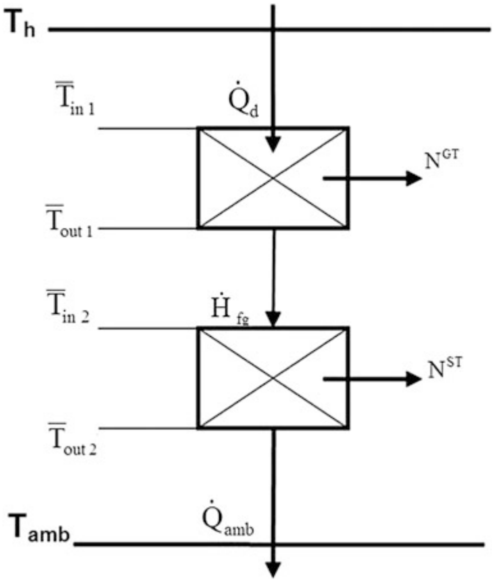
The stream of heat output from an i-th cycle (engine) by means of entropy averaged temperatures can only be expressed by streams of heat $\dot{Q}_{in\,1} \equiv \dot{Q}_d$ delivered into the system from the source with the temperature $\overline{T}_{out\,0} \equiv T_h$

$$
\dot{Q}_{out\,i} = \dot{Q}_{in\,i+1} = \dot{Q}_d \prod_{n=1}^{i} \frac{\overline{T}_{out\,n}}{\overline{T}_{in\,n}}. \tag{2.11}
$$

From the Eq. (2.11) we obtain the relation for the heat output from the system into the environment

$$
\dot{Q}_{amb} = \dot{Q}_d \prod_{i=1}^{j} \frac{\overline{T}_{out\,i}}{\overline{T}_{in\,i}}. \tag{2.12}
$$

Fig. 2.4 Diagram of hierarchical, 2-cycle, gas–steam heat engine



By applying the relation (2.11) to the equation in (2.9) the value of the total power of the system can be defined as:

$$N = \sum_{i=1}^{j} N_i = \dot{Q}_d \left(1 - \prod_{i=1}^{j} \frac{\overline{T}_{out\ i}}{\overline{T}_{in\ i}} \right). \tag{2.13}$$

The value in the brackets on the right hand side of Eq. (2.13) denotes the energy efficiency η_{1-j} of generating power in a system with *j*-cycles expressed by entropy averaged temperatures:

$$\eta_{1-j} = 1 - \prod_{i=1}^{j} \frac{\overline{T}_{out\ i}}{\overline{T}_{in\ i}}. \tag{2.14}$$

For instance, for a two-cycle system this efficiency, by additionally using relation (2.10), can be expressed by the equation:

$$\eta_{1-2} = 1 - \frac{\overline{T}_{out\ 1}}{\overline{T}_{in\ 1}} \frac{\overline{T}_{out\ 2}}{\overline{T}_{in\ 2}} = \eta_1 + \eta_2 - \eta_1 \eta_2. \tag{2.15}$$

The final form of the Eq. (2.8) which distinguishes the location of the origin of exergy losses in the system makes it possible to find ways of its thermodynamic improvement. It indicates the places of greatest exergy losses which determine its low effectiveness, and therefore, indicates the places in which it could be improved. It also indicates the entropy averaged temperatures and direction of altering its values so as to improve the thermodynamic perfection of the system.

In addition, the presented equation makes the quantitative analysis of the reasons which increase this perfection. Furthermore, the analysis of the variability of parameters of the preceding cycles is possible resulting in a change to exergy losses in the subsequent phases, thereby affecting exergy losses in the whole system. Thus, it is possible to undertake the justification for these changes.

The final form of the Eq. (2.8) makes it possible to analyze the effect of energy efficiency of the particular engines (entropy averaged temperatures of circulating media) on the total energy efficiency of the system.

For the case of a two-cycle system ($j = 2$), (Figs. 2.1, 2.4), the exergy losses as a result of irreversible heat flow between the sources and circulating media (Eq. 2.8), the total power of the system (Eq. 2.9) and the energy efficiency can be expressed by the subsequent equations:

- exergy losses

$$\delta \dot{B}^{G-S} = T_{amb}\left(\dot{Q}_d \frac{T_h - \overline{T}_{in\,1}}{T_h \overline{T}_{in\,1}} + \dot{H}_{fg} \frac{\overline{T}_{out\,1} - \overline{T}_{in\,2}}{\overline{T}_{out\,1}\overline{T}_{in\,2}} + \dot{Q}_{amb} \frac{\overline{T}_{out\,2} - T_{amb}}{\overline{T}_{out\,2} T_{amb}}\right) \tag{2.16}$$

- power of the system

$$N^{G-S} = N^{GT} + N^{ST} = \dot{Q}_d \frac{\overline{T}_{in\,1} - \overline{T}_{out\,1}}{\overline{T}_{in\,1}} + \dot{H}_{fg} \frac{\overline{T}_{in\,2} - \overline{T}_{out\,2}}{\overline{T}_{in\,2}} = \dot{Q}_d \eta_{GT} + \dot{H}_{fg} \eta_{ST} \tag{2.17}$$

or by applying the final form of the Eq. (2.9)

$$\begin{aligned} N^{G-S} &= N_C - \delta \dot{B}^{G-S} \\ &= \dot{Q}_d \frac{T_h - T_{amb}}{T_h} - T_{amb}\left(\dot{Q}_d \frac{T_h - \overline{T}_{in\,1}}{T_h \overline{T}_{in\,1}} + \dot{H}_{fg} \frac{\overline{T}_{out\,1} - \overline{T}_{in\,2}}{\overline{T}_{out\,1}\overline{T}_{in\,2}} + \dot{Q}_{amb} \frac{\overline{T}_{out\,2} - T_{amb}}{\overline{T}_{out\,2} T_{amb}}\right) \end{aligned} \tag{2.18}$$

- energy efficiency (compare Eqs. 2.10, 2.15)

$$\eta_{G-S} = \frac{N^{GT} + N^{ST}}{\dot{E}_{ch}^{gas}} = \eta_{GT} + \eta_{ST} - \eta_{GT}\eta_{ST} \tag{2.19}$$

where:

$\dot{E}_{ch}^{gas} \equiv \dot{Q}_d$	stream of chemical energy of the gas combustion in the gas engine,
N^{GT}, N^{ST}	power of the gas and steam engines,
η_{GT}, η_{ST}	energy efficiency of the gas and steam engines,
$\overline{T}_{in\,1}$, $\overline{T}_{out\,2}$	subsequent mean thermodynamic temperature in the combustion chamber of the gas turbine and temperature of steam saturation in the condenser of the steam turbine,
$\overline{T}_{out\,1}$, $\overline{T}_{in\,2}$	subsequent mean thermodynamic temperature of flue gas and steam in the heat recovery steam generator.

Only the value of temperature $\overline{T}_{in\,2}$ can be determined by the designer by means of altering the number of the heating surfaces, as well as their design and sizes, their location as well by means of adopting temperature intervals, i.e. differences between the temperature of flue gas and water and steam in the process heat exchange in the waste boiler. Concurrently, the values of temperatures $\overline{T}_{in\,1}$, $\overline{T}_{out\,1}$ are relative only to the type of turbogenerator used in a given system and there is no way of affecting them, as well as no effect that can be possibly made to temperature $\overline{T}_{out\,2}$, which is relative to ambient temperature.

The expression (compare Eq. 2.3)

$$T_{amb}\dot{H}_{fg}\frac{\overline{T}_{out\,1}-\overline{T}_{in\,2}}{\overline{T}_{out\,1}\overline{T}_{in\,2}} \tag{2.20}$$

in Eq. (2.18) denotes the loss of exergy stream in the heat recovery steam generator $\delta\dot{B}_{HRSG}$ (compare 6.2). The higher the temperature $\overline{T}_{in\,2}$, which can be determined by the designer, the smaller the losses and, thereby, the greater the electrical capacity of the steam turbogenerator (Eq. 2.17)

$$N^{ST}=\dot{H}_{fg}\frac{\overline{T}_{in\,2}-\overline{T}_{out\,2}}{\overline{T}_{in\,2}}=\dot{H}_{fg}\eta_{ST}. \tag{2.21}$$

To summarize, the structure of the heat recovery steam generator should be adopted in a way that ensures that the equation in (2.20) assumes the lowest possible value. The decrease in the value of (2.20), which means the reduction of exergy losses in a repowered unit, the greater the increase in the production of electricity in a unit. However, investment required for the power unit is greater, which can result in the limitation of the economic efficiency of the operation of the repowered unit (Eq. 6.3). Therefore, there is a technical and economic optimum, which has to be sought.

Reference

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